TEACHERS FORUM®

QUESTION BANK

(SOLVED)

KERALA STATE

+2 MATHEMATICS

CONTENTS

1 RELATIONS AND FUNCTIONS

PREVIOUS YEARS' QUESTIONS AND ANSWERS

1. Which of the following relations on A = {1, 2, 3} is an equivalence relation ?

   (a) {(1, 1), (2, 2), (3, 3)} (b) {(1, 1), (2, 2), (3, 3), (1, 2)} **(2022)**

   (c) {(1, 1), (3, 3), (1, 3), (3, 1)} (d) None of these

Ans. (a) { (1,1), (2,2), (3,3) }

22. R = {(x, y) : x, y $\notin$ Z, (x – y) is an integer}. Show that R is an equivalence relation

Ans. For any a $\in$ Z , a – a = 0 is an integer. **(2022)**

Therefore R is reflexive.

Difference between two integers is also an integer.

That is if x – y is an integer ,then y – $x$is an integer. So R is symmetric.

if x – y , and y – z are integers, then x – z is also an integer. So R is transitive.

Therefore R is an equivalence relation.

2. If * is a binary operation on R defined by a * b = $\frac{ab}{3}$

   (a) Find the identity element of *. **(2022)**

   (b) Find the inverse of 3.

Ans.(a) Let e be the identity element of a.

Then a * e = e * a = a

a * e = a $\Rightarrow \frac{ae}{3}$ = a $\Rightarrow$ e = 3

(b) Let a^{-1} be the inverse of a.

Then a^{-1} * a = a * a^{-1} =e

a * a^{-1} = e $\Rightarrow \frac{a.a^{-1}}{3} = 3$ $a^{-1} = \frac{9}{a}$

inverse of 3 is, $3^{-1} = \frac{9}{3} = 3$

3. (a) Discuss the continuity of the function

$$f(x) = \begin{cases} 3x + 1, & \text{if } x \leq 3 \\ x^2 + 1, & \text{if } x > 3 \end{cases}$$

(b) Verify Rolle's theorem for the function $f(x) = 2x^2 - 12x + 1$ in [2, 4]. **(2022)**

Ans. (a) LHL = $\lim_{x+3} (3x + 1) = 10$

RHL = $\lim_{x+3} (x^2 + 1) = 10$

f (3) = 10

LHL = RHL = f(x)

Therefore f(x) is continuous .

(b) f(x) is continuous on [2, 4]

f(x) is differentiable on (2, 4)

f(a) = f(2) = -15

f(b) = f(4) = -15

here f(a) = f(b)

f' (x) = 4 x − 12

f' (c) = 0 $\Rightarrow$ 4 c − 12 = 0

$\Rightarrow$ c = 3 $\in$ (2, 4) Hence verified.

4. (i) Let R be a relation on a set A = {1, 2, 3}, defined by R = {(1, 1), (2, 2), (3, 3), (1, 3)}. Then the ordered pair to be added to R to make it a smallest equivalence relation is ____________.

(a) (2, 1) (b) (3, 1) (c) (1, 2) (d) (1, 3)

(ii) Determine whether the relation R in the set A = {1, 2, 3, 4, 5, 6} as R = {(x, y) : y is divisible by x } is reflexive, symmetric and transitive. **(2021)**

Ans. (i) (b) (3, 1)

(ii) R = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 4), (2, 6), (3, 6) }

(a, a) $\in$ R for all a $\in$ A.

$\therefore$ R is reflexive

(1, 2) $\in$ R but (2, 1) $\notin$ R.

$\therefore$ R is not symmetric

If (a, b) $\in$ R and (b, c) $\in$ R.

then (a, c) $\in$ R for a $\in$ A

$\therefore$ R is transitive.

$\therefore$ R is reflexive and transitive but not symmetric.

5. (i) Let f : {1, 3, 4} $\rightarrow$ {1, 2, 5} and g : {1, 2, 5} $\rightarrow$ {1, 3} be given by

f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down go f.

(ii) Consider f : R $\rightarrow$ R given by f(x) = 2x + 1. Show that f is invertible.

Find the inverse of f. **(2021)**

Ans. (i) gof(1) = g(f(1)) = g(2) = 3

gof(3) = g(f(3)) = g(5) = 1

gof(4) = g(f(4)) = g(1) = 3

(ii) Let $y = 2x + 1$

$2x = y - 1$

$$x = \frac{y-1}{2}$$

g is the inverse of f if,

fog = gof

Let $g(x) = \frac{x-1}{2}$

$$fog(x) = f(g(x)) = f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right) + 1 = x - 1 + 1 = x$$

$$gof(x) = g f((x)) = g(2x+1) = \frac{2x+1-1}{2} = \frac{2x}{2} = x$$

gof(x) = fog(x) = x.

$\therefore$ f is invertible

$\therefore$ $f^{-1}(x) = \frac{x-1}{2}$

6. (i) Let R be a relation in the set N of natural numbers given by

R = {(a, b) : a = b -2}. Choose the correct answer.

(a) (2, 3) $\in$ R (b) (3, 8) $\in$ R (c) (6, 8) $\in$ R (d) (8, 7) $\in$ R

(ii) Let * be a binary operation defined on the set Z of integers a * b = a + b + 1. Then find the identity element. **(2020)**

Ans. (i) (c) (6, 8) $\in$ R

(ii) a * e = a

a + e + 1 = a

e + 1 = 0 $\Rightarrow$ e = -1

7. Let A = **R** - {3} and B : **R** - {1}. Consider the function f : A $\rightarrow$ B defined by $f(x) = \frac{x-2}{x-3}$

(i) Is f one-one and onto? Justify your answer. **(2020)**

(ii) Is it invertible? Why?

(iii) If invertible, find inverse of $f\ (x)$

Ans. (i) $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$(x_1 - 2)\ (x_2 - 3)\ = (x_2 - 2)\ (x_1 - 3)$

$x_1x_2 - 3\ x_1 - 2x_2 + 6 = x_1x_2 - 2\ x_1 - 3x_2 + 6$

$-3\ x_1 - 2\ x_2 = -2\ x_1 - 3\ x_2$

$-3\ x_1 + 2\ x_1 = 2\ x_2 - 3\ x_2$

$\Rightarrow \qquad -x_1 = -\ x_2$

$\Rightarrow \qquad x_1 = x_2 \quad \therefore f$ is one - one.

Now $\quad y = \frac{x - 2}{x - 3}$

$yx - 3y \qquad = x - 2$

$(y - 1)\ x \qquad = 3y - 2$

$x = \frac{3y - 2}{y - 1} \in A \qquad \therefore f$ is onto

(ii) Yes. Because it is bijective.

(iii) $f^{-1}\ (x) = \frac{3x - 2}{x - 1}$

8. (a) If f(x) = sin x, g(x) = x^2, $x \in$ R ; then find (fog) (x) **(2019)**

. (b) Let u and v be two functions defined on [R as u (x) : 2x - 3 and v(x) = $\frac{3 + x}{2}$ that u and v are inverse to each other.

Ans. (a) f(x) = sin x, g(x) = x^2

(fog) (x) = f (g (x)) = f (x^2) = sin x^2

(b) (u.v) x = u (v (x)) = u $\left(\frac{3 + x}{2}\right)$ = 2 $\left(\frac{3 + x}{2}\right)$ - 3 = x

(c) (v.u) x = v (u (x)) = v (2x - 3) = $\left(\frac{3 + 2x - 3}{2}\right)$ = x

9. (a) The function P is defined as "To each person on the earth is assigned a date of birth". Is this function one-one ? Give reason. **(2019)**

(b) Consider the function, f : $\left[0, \frac{\pi}{2}\right] \longrightarrow$ R

given by f (x) = sin x and g : $\left[0, \frac{\pi}{2}\right] \longrightarrow$ R

given by g (x) = cos x.

(i) Show that f and g are one-one functions.

(ii) Is f + g one - one? Why?

(c) The number of one-one functions from a set containing 2 elements to a set containing 3 elements is _________.

(i) 2 (ii) 3 (iii) 6 (iv) 8

Ans. (a) Not One - One

Because different persons have same birthday.

(b) f (x) = sin x

(i) f (x_1) = f (x_2) $\Rightarrow$ sin $x_1 =$ sin x_2

$\Rightarrow$ x_1 = x_2 $\Rightarrow$ f is One - One

g (x) = cos x

g (x_1) = g (x_2) $\Rightarrow$ cos $x_1 =$ cos x_2

$\Rightarrow$ x_1 = x_2 $\Rightarrow$ g is One - One

(ii) (f + g) (x) = sin x + cos x

(f + g) (x_1) = (f + g) (x_2)

$\Rightarrow$ sin x_1 + cos x_1 = sin x_2 + cos x_2

$\Rightarrow$ sin x_1 − sin x_2 = cos x_2 - cos x_1

$\Rightarrow x_1 = x_2$ $\Rightarrow$ g is One - One

$\Rightarrow$ $\cos \frac{x_1 + x_2}{2} = \sin \frac{x_1 + x_2}{2}$.

$\Rightarrow$ $x_1 = \frac{\pi}{2} - x_2$

$\Rightarrow$ f + g is not one-one

(c) (iii) 6

10. If $f(x) = \frac{x}{x\text{-}1}$, $x \neq 1$

(a) Find fof (x) (b) Find the inverse of f . **(2018)**

Ans. (a) $f(x) = \frac{x}{x\text{-}1}$, $x \neq 1$

$$\text{fof}(x) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{x}{1} = x$$

(b) Since fof (x) = x

$$f^{-1} = \frac{y}{y-1}, \ y \neq 1$$

11. Let A = N × N and '*' be a binary operation on A defined by (a,b)* (c,d) = (a+c,b+d)

(a) Find (1,2) * (2,3) **(2018)**

(b) Prove that '*' is commutative. (c) Prove that '*' is associative.

Ans. (a) (a, b) * (c, d) = (a + c, b + d)

(1, 2) *(2, 3) = (1 + 2, 2 + 3) = (3, 5)

(b) (a, b) * (c, d) = (a + c, b + d)

(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)

$\Rightarrow$ * is commutative

(c) (a, b) * [(c, d) * (e,f)] = (a,b) * [(c + d) , (d + f)] = (a + c + e, b + d + f)

[(a,b) * (c,d) * (e,f) = (a+c , b+d) * (e,f)] = (a + c + e, b + d + f)

i.e (a,b) * [(c,d) * (e,f)] = [(a,b) * (c,d)] * (e,f)

$\therefore$ * is associative

12. (a) Let R be a relation defined on A = { 1, 2, 3} by R = { (1,3) , (3,1) ,(2,3)} R is

a. Reflexive b. Symmetric c. Transitive d. Reflexive but not transitive

(b) Find fog and gof if f(x) = | x +1| and g(x) = 2x -1.

(c) Let * be a binary operation defined on N × N by

(a,b) * (c,d) = (a+c ,b+d).

Find the identity element for * if it exists. **(2017)**

Ans. (a) Symmetric

(b) fog = $f\left(g(x)\right) = f(2x - 1) = \left|2x - 1 + 1\right| = \left|2x\right| = 2x$

gof= $g\left(f(x)\right) = g\left(\left|x + 1\right|\right) = 2\left|x + 1\right| - 1$

(c) Let (e, f) be the identity function .

then $(a, b) * (e, f) = (a + e, b + f)$

For identify function $a + e \Rightarrow e = 0$

and $b + f = b \Rightarrow f = 0$

Identify element does not exist.

13. (a) If R = $\left\{(x, y): x, y \in Z, x - y \in Z\right\}$, then the relation R is

(i) Reflexive but not transitive (ii) Reflexive but not symmetric

(iii) Symmetric but not transitive (iv) an Equivalence relation

(b) Let * be a binary operation on the set Q of rational numbers by $a * b = 2a + b$.

Find 2 * (3 * 4) and (2 * 3) * 4

(c) Let f : R $\rightarrow$ R, g : R$\rightarrow$ R be two one-one functions. Check whether go$f$ is one-one or not. **(2017)**

Ans. (a) (iv) an Equivalence relation

(b) 2 *(3 * 4) = 2 *(2 x 3 + 4) = 2 * 10 = 2 x 2 + 10 = 14

$(2 * 3) * 4 = (2 \times 2 + 3) * 4 = 7 * 4 = 2 \times 7 + 4 = 18$

(c) $(gof)(x_1) = (gof)(x_2)$

$\Rightarrow g[f(x_1)] = g[f(x_2)]$

$\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$ $\Rightarrow$ gof is one-one.

14. (a) The function given $f : N \rightarrow N$, by $f(x) = 2x$ is

(i) one-one and onto (ii) one-one but not onto

(iii) not one-one and not onto (iv) onto, but not one-one

(b) Find gof(x), if $f(x) = 8x^3$ and $g(x) = x^{1/3}$

(c) Let $*$ be an operation such that

a $*$ b = LCM of a and b defined on the set A = {1,2,3,4,5}. Is $*$ binary operation?

Justify your answer. **(2016)**

Ans. (a) (ii) f is one-one but not onto.

(b) $gof(x) = g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 2x$

(c) a * b = LCM (a,b)

2 * 3 = LCM (2, 3) = 6 $\notin$ A

5 * 2 = LCM (5, 2) = 10 $\notin$ A

So * is not a binary operation.

15. (a) If $f : R \rightarrow R$ and $g : R \rightarrow R$ defined by $f(x) = x^2$ and $g(x) = x + 1$, then gof(x) is

(i) $(x + 1)^2$ (ii) $x^3 + 1$ (iii) $x^2 + 1$ (iv) $x + 1$ **(2016)**

(b) Consider the function $f : N \rightarrow N$, given by $f(x) = x^3$. Sow that the function f is injective but not surjective.

(c) The given table shown an operation $*$ on A {p, q}

$*$	p	q
p	p	q
q	p	q

(i) Is $*$ a binary operation on A? (ii) Is $*$ commutative? Give reason.

Ans. (a) $gof(x) = g(f(x)) = g(x^2) = x^2 + 1$

(b) $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1^3 - x_2^3 = 0$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1 x_2 + x_2^2) = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2$

$\therefore f$ is injective, i.e., one-one

Surjective

Let $y = 4 \in N$,

$\Rightarrow f(x) = 4 \Rightarrow x^3 = 4$

$\Rightarrow x = 4^{1/3} \notin N$,

$\therefore f$ is not surjective, i.e., onto

(c) (i) Yes.

(ii) No

$p * q = q$ and $q * p = p$

Since $p * q \neq q * p$, * is not commutative

16. (a) What is the minimum number of ordered pairs to form a non-zero reflexive relation on a set of n elements?

(b) On the set R of real numbers, S is a relation defined as S = {(x, y) | $x \in$ R, y $\in$ R, x + y = xy} . Find a $\in$ R such that 'a' is never the first element of an ordered pair is S. Also find b $\in$ R such that 'b' is never the second element of an ordered pair is S.

(c) Consider the function $f(x) = \dfrac{3x+4}{x-2}$, $x \neq 2$. Find a function g(x) on a suitable domain such that (go$f$) (x) = x= ($f$og) (x) **(2015)**

Ans. (a) n

(b) $a + b = ab \Rightarrow ab - b = a$

$\Rightarrow b(a-1) = a$

$\Rightarrow b = \dfrac{a}{a-1}$

$\Rightarrow b \neq 1$ Similarly; $a \neq 1$

(c) $y = \dfrac{3x+4}{x-2}$

$\Rightarrow 3x + 4 = y(x-2)$

$\Rightarrow 3x + 4 = yx - 2y \Rightarrow yx - 3x = 2y + 4$

$\Rightarrow x = \dfrac{2y+4}{y-3}$ $\Rightarrow g(x) = \dfrac{2x+4}{x-3}$

17. (a) Let R be the relation on the set N of the natural numbers given by

R = {(a, b): a - b > 2, b > 3}. Choose the correct answer

(A) $(4,1) \in R$ (B) $(5, 8) \in R$ (C) $(8, 7) \in R$ (D) $(10, 6) \in R$

(b) If $f(x) = 8x^3$ and $g(x) = x^{1/3}$. Find $g(f(x))$ and $f(g(x))$

(c) Let $*$ be a binary operation on the set Q of rational numbers defined by $a * b = \frac{ab}{3}$. Check whether $*$ is commutative and associative? **(2014)**

Ans. (i) (d) $(10, 6) \in R$

(ii) Given ; $f(x) = 8x^3$ and $g(x) = x^{1/3}$

$$g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 2x$$

$$f(g(x)) = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$$

(iii) $a * b = \frac{ab}{3} = \frac{ba}{3} = b * a$. $\Rightarrow$ * is commutative.

$$a * (b * c) = a * \frac{bc}{3} = \frac{abc}{9}$$

$$(a * b) * c = \frac{ab}{3} * c = \frac{abc}{9}$$

$\Rightarrow a * (b * c) = (a * b) * c$. $\Rightarrow$ * is associative.

18. Consider $f : R \to R$ given by $f(x) = 5x + 2$.

(a) Show that f is one-to-one.

(b) Is f invertible? Justify your answer.

(c) Let $*$ be a binary operation on N defined by $a * b$ = HCF of a and b.

(i) Is $*$ commutative? (ii) Is $*$ associative? **(2013)**

Ans. (a) $f(x_1) = f(x_2)$

$\Rightarrow 5x_1+2 = 5x_2 + 2$

$5x_1 = 5x_2$.

$\Rightarrow x_1 = x_2$ .i.e $f(x)$ is one - one

(b) Let $y = f(x)$.

$\Rightarrow x = \frac{y-2}{5} \in R$

$$f(x) = 5\left(\frac{y-2}{5}\right) + 2$$

$= y - 2 + 2 = y$.

$\Rightarrow f$ is onto

$\Rightarrow f$ a bijective function and f is invertible

(c) (i) a* b = H.C.F of a and b = H.C.F of b and a = b * a

$\Rightarrow$ * is commutative

(ii) a*(b*c) = a* (HCF b c) = HCF(a,b,c)

(a*b) *c = (HCF a b)* c = HCF (a,b,c)

$\Rightarrow$ * is commutative.

19. (a) Give an example of a relation on a set A = {1,2,3,4}, which is reflexive, symmetric but not transitive.

(b) Show that $f : [-1, 1] \to R$ is given by $f(x) = \frac{x}{x+2}$ is one-one.

(c) Let $*$ be a binary operation on Q^+ defined by $a * b = \frac{ab}{6}$. Find inverse of 9 with respect to $*$. **(2013)**

Ans. (a) A = {1,2,3,4 }

R = {(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (2,3), (3,2), (3,4), (4,3)}

(b)

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$$

$$\Rightarrow x_1(x_2+2) = x_2(x_1+2)$$

$$\Rightarrow x_1x_2+2x_1 = x_1x_2+2x_2.$$

$$\Rightarrow 2x_1 = 2x_2.$$

$\Rightarrow x_1 = x_2$ i.e, $f(x)$ is one to one

(c) a * e = a, where e is the identity element

$$\Rightarrow \frac{ae}{6} = a \Rightarrow e = 6$$

If b is the inverse of 9, then 9 * b = e

$$\Rightarrow \frac{9b}{6} = 6$$

$$\Rightarrow b = 4$$

ie, inverse of 9 w.r.t * is 4.

Additional Questions and Answers

1. Let R be a relation on the set A = {1,2,3,4,5,6} defined as $R = \{(x, y) : y = 2x - 1\}$

(i) Write Rin roster form and find it's domain and range

(ii) Is R an equivalence relation? Justify

Ans. (i) $R = \{(1, 1), (2, 3), (3, 5)\}$

Domain = {1, 2, 3}; Range = {1, 3, 5}

(ii) Since (2, 2) $\notin$ R, R is not reflexive

(2, 3) $\in$ R but (3, 2) $\notin$ R; R is not symmetric

(2, 3) $\in$ R, (3, 5) $\in$ R but (2, 5) $\notin$ R, R is not transitive

$\therefore$ R is not an equivalence relation

2. The relation R defined on the A = {−1,0,1} as R = {(a,b): a^2 = b}

(i) Check whether R is reflexive, symmetric and transitive

(ii) Is R an equivalence relation?

Ans. (i) (-1, -1) $\notin$ R, R is not reflexive

(-1, 1) $\in$ R and (1, -1) $\notin$ R, R is not symmetric

(-1, 1) $\in$ R, (1, 1) $\in$ R and (-1, 1) $\in$ R, R is transitive.

(ii) R is not reflexive, not symmetric and transitive.

So R is not an equivalence relation

3. Let A = {1, 2, 3}. Give an example of a relation on A which is

(i) Symmetric but neither reflexive nor transitive

(ii) Transitive but neither reflexive nor symmetric

Ans. (i) R = $\{(1, 2), (2, 1)\}$

(1, 1) $\notin$ R $\Rightarrow$ R is not reflexive

(1, 2) $\in$ R $\Rightarrow$ (2, 1) $\in$ R, R is symmetric

(1, 2) $\in$ R, (2, 1) $\in$ R but (1, 1) $\notin$ R, R is not transitive

(ii) R = $\{(1, 2), (1, 3), (2, 3)\}$

(1, 1) $\notin$ R $\Rightarrow$ R is not reflexive

(1, 2) $\in$ R but (2, 1) $\notin$ R, R is not symmetric

(1, 2) $\in$ R,(2, 3) $\in$ R $\Rightarrow$ (1, 3) $\in$ R, R is transitive

4. (i) Let f be a function defined by $f(x) = \sqrt{x}$ is a function if it defined from

(f : N $\to$ N, f : R $\to$ R, f : R $\to$ R^+, f : $R^+ \to R^+$)

(ii) Check the injectivity and surjectivity of the following functions

(a) f : N $\to$ N given by $f(x) = x^3$ (b) f : R $\to$ R given by $f(x) = [x]$

Ans. (i) $f : R^+ \to R^+$ For x,y $\in$ N

(ii) (a) $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y \Rightarrow$ f is injective

For 2 $\in$ N, there does not exist x in the domain N such that $f(x) = x^3 = 2$.

$\therefore f$ is not surjective

(b) f : R $\to$ R given by $f(x) = [x]$

It seen that $f(1.1) = 1$ and $f(1.8) = 1$;

But 1.1 ≠ 1.8; $\therefore f$ is not injective

There does not exist any element $x \in$ R such that $f(x) = 0.7$

$\therefore f$ is not surjective

5. (a) Find fog and gof if

(i) $f(x) = |x|$ and $g(x) = |3x + 4|$ (ii) $f(x) = 16x^4$ and $g(x) = x^{1/4}$

(b) If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, prove that $f o f(x) = x$

Ans. (a) (i) $f(x) = |x|$ and $g(x) = |3x + 4|$

$$\Rightarrow f o g(x) = f(g(x)) = f(|3x+4|) = \big||3x+4|\big| = |3x+4|$$

$$g o f(x) = g(f(x)) = g(|x|) = |3|x| + 4|$$

(ii) $$f o g(x) = f(g(x)) = f(x^{1/4}) = 16(x^{1/4})^4 = 16x$$

$$g o f(x) = g(f(x)) = g(16x^4) = (16x^4)^{1/4} = 4x$$

(b) $$f o f(x) = f(f(x)) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

6. Let S $= \{(1, 2), (2, 3), (3, 4)\}$

(i) Find the domain and range of S (ii) Find S^{-1}

(iii) Find the domain and range of S^{-1}

(iv) Verify that S^{-1} is a function using the graph of S and S^{-1}

Ans. (i) Domain = {1, 2, 3}; Range = {2, 3, 4}

(ii) $S^{-1} = \{(2, 1), (3, 2), (4, 3)\}$

(iii) Domain = {2, 3, 4}; Range = {1, 2, 3}

(iv) Yes, S^{-1} is a function because x coordinates do not intersect

7. (i) Consider $f : \{3, 4, 5, 6\} \to \{8, 10, 12, 13, 14\}$ and $x = \{(3, 8), (4, 10), (5, 12), (6, 14)\}$. State whether f has inverse? Give reason

(ii) Consider $f : R \to R$ given by $f(x) = 3x + 2$. Show that f is invertible. Find the inverse of f

Ans. (i) Distinct elements in set {3, 4, 5, 6} has distinct images nuder f. $\therefore f$ is one-one

But 13 in the codomain has no pre image. $\therefore f$ is not onto.

$\therefore f$ has no inverse

(ii) $$f(x) = 3x + 2; \text{ then}$$

$$f(x_1) = f(x_2) \Rightarrow 3x_1 + 2 = 3x_2 + 2 \Rightarrow x_1 = x_2$$

Hence F is one - one

For $y \in R$, let $y = 3x + 2 \Rightarrow x = \dfrac{y-2}{3} \in R$

$$f(x) = f\left(\frac{y-2}{3}\right) = 3\left(\frac{y-2}{3}\right) + 2 = y \Rightarrow f \text{ is onto}$$

$g : R \to R$ such that $g(y) = \dfrac{y-2}{3}$

$$gof(x) = g(f(x)) = g(3x + 2) = \frac{3x + 2 - 2}{3} = x$$

$$fog(y) = f(g(y)) = f\left(\frac{y - 2}{3}\right) = 3\left(\frac{y - 2}{3}\right) + 2 = y$$

8. Choose the correct answer from the bracket

If $x \neq 1$ and $f(x) = \frac{x + 1}{x - 1}$ is a real function, then $fof(2)$ = ---------

(1, 2, 3, 4)

(i) What is the inverse of f (ii) Find $f(3) + f^{-1}(3)$

Ans. (i) 2

(ii) Let g: range of $f \rightarrow R - \{1\}$ be the inverse of f

Let y be any arbitrary element in the range of f, then $y = f(x) = \frac{x + 1}{x - 1}$

$$\Rightarrow xy - y = x + 1 \Rightarrow x(y - 1) = y + 1 \Rightarrow x = \frac{y + 1}{y - 1}, x \neq 1$$

g: range of $f \rightarrow R - \{1\}$ as $g(y) = \frac{y + 1}{y - 1}$

$$gof(x) = g(f(x)) = g\left(\frac{x + 1}{x - 1}\right) = \frac{\frac{x + 1}{x - 1} + 1}{\frac{x + 1}{x - 1} - 1} = x$$

$$\therefore f^{-1} = g \Rightarrow f^{-1}(y) = \frac{y + 1}{y - 1}, y \neq 1$$

(iii) $f(3) = 2,\ f^{-1}(3) = 2$

$$\Rightarrow f(3) + f^{-1}(3) = 2 + 2 = 4$$

9. (i) Determine whether the following is a binary operation or not? Justify

$a * b = 2^a b$ defined on Z

(ii) Determine whether * is commutative or associative if

$$a * b = \frac{ab}{6}, a, b \in R$$

Ans. (i) $a * b = 2^a b$

If a is negative, then 2^a becomes a fraction

Eg : $-1*3 = 2^{-1} .3 = \frac{3}{2} \notin Z$; $\therefore$ * is not a binary operation

(ii) $a * b = \frac{ab}{6} \Rightarrow b * a = \frac{ba}{6} = \frac{ab}{6} = a * b$

$\Rightarrow$ * is commutative

$$(a * b) * c = \frac{\frac{ab}{6} . c}{6} = \frac{abc}{36} \Rightarrow a * (b * c) = \frac{a.\frac{bc}{6}}{6} = \frac{abc}{36}$$

$\therefore$ a * (b * c) = a * (b * c) $\Rightarrow$ * is associative

10. Consider the binary operation * :Q $\rightarrow$ Q where Q is the set of rational numbers is defined as a * b = a + b − ab

(i) Find 2 * 3

(ii) Is identity for * exist? If yes, find the identity element

(iii) Are elements of Q invertible? If yes, find the inverse of an element in Q.

Ans. (i) $2 * 3 = 2 + 3 - 6 = -1$

(ii) $a * e = a + e - ae = a \Rightarrow e - ae = 0$

$\Rightarrow e(1 - a) = 0 \Rightarrow e = 0$

$\Rightarrow e * a = a$

$\Rightarrow e + a - ea = a$

$\Rightarrow e - ea = 0$

$\Rightarrow e = 0$ is the identity element

(iii) $a * a^{-1} = a + a^{-1} - aa^{-1} = 0$

(iv) $\Rightarrow a^{-1}(1 - a) = -a \Rightarrow a^{-1} = \frac{-a}{1 - a} = \frac{a}{a - 1}$

11. The binary operation $*$: R x R $\rightarrow$ R is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$.

Ans. 18

12. State the reason for the relation R in the set {1, 2, 3} given by R= {(l, 2), (2, 1)} not to be transitive.

Ans. $(1, 2) \in R$, $(2, 1) \in R$ but $(1, 1) \notin R$

13. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not.

Ans. f is a one-one function

14. If the binary operation * on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value of 2 * 4.

Ans. 50

15. Let * be a binary operation defined by a * b = 2a + b – 3. Find 3 * 4.

Ans. $3 * 4 = 2 \times 3 + 4 - 3 = 7$

16. Prove that if E and F are independent events, then the events E and F’ are also independent.

Ans.

$$P(E \cap F') = P(E) - P(E \cap F)$$

$$= P(E) - P(E) . P(F)$$

$$= P(E)[1 - P(F)] = P(E)P(F')$$

17. A binary operation * is defined on the set x = R – { – 1 } by

$x * y = x + y + xy, \forall x, y \in X$.

Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

Ans. (i) commutative : let $x, y \in R - \{-1\}$ then

$x * y = x + y + xy = y + x + yx = y * x$

$\therefore$ * is commutative

(ii) Associative : let $x, y, z \in R - \{-1\}$ then

$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x(y + z + yz)$

$= x + y + z + xy + yz + zx + xyz$

$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy) . z$

$= x + y + z + xy + yz + zx + xyz$

$x * (y * z) = (x * y) * z$ $\therefore$ * is Associative

(iii) Identity Element : let $e \in R - \{-1\}$ such that $a * e = e * a = a \; \forall a \in R - \{-1\}$

$\therefore a + e + ae = a \Rightarrow e = 0$

(iv) Inverse : let $a * b = b * a = e = 0$; $a, b \in R - \{-1\}$

$\Rightarrow a + b + ab = 0$

$\therefore b = \frac{-a}{1+a}$ or $a^{-1} = \frac{-a}{1+a}$

18. If f, g : R $\to$ R be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$.

Then find fog and gof. Hence find fog(–3), fog(5) and gof (–2).

Ans. $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$

$(fog)(x) = f(g(x)) = ||x| - 1| + |x| - x$

$(gof)(x) = g(f(x)) = ||x| + x| - |x| - x$

$(fog)(-3) = 6$, $(fog)(5) = 0$, $(gof)(-2) = 2$

19. Let A = R x R and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Prove that $*$ is commutative and associative. Find the identity element for * on A. Also write the inverse element of the element (3, – 5) in A.

Ans. $\forall a, b, c, d, e, f \in \Re$

$((a, b) * (c, d) * (e, f) = (a + c, b + d) * (e, f)$

$= (a + c + e, b + d + f) \quad \to (3)$

$$(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f)$$

$$= (a + c + e, b + d + f) \quad \rightarrow (4)$$

$*$ is Associative

Let (x, y) be on identity element in $\Re \times \Re$

$$\Rightarrow (a, b) * (x, y) = (a, b) = (x, y) * (a, b)$$

$$\Rightarrow a + x = a, b + y = b$$

$$x = 0 , y = 0$$

$\therefore$ (0, 0) is identity element

Let the inverse element of (3, – 5) be (x_1, y_1)

$$\Rightarrow (3, -5) * (x_1, y_1) = (0, 0) = (x_1, y_1) * (3, -5)$$

$$3 + x_1 = 0, \; -5 + y_1 = 0$$

$$\Rightarrow x_1 = -3 , y_1 = 5$$

$\Rightarrow$ (– 3, 5) is an inverse of (3, – 5)

20. If $f(x) = \sqrt{x^2 + 1}$; $g(x) = \dfrac{x - 1}{x^2 + 1}$ and $h(x) = 2x - 3$, then find $f^{|}[h^{|}\{g^{|}(x)\}]$.

Ans. $$f(x) = \sqrt{x^2 + 1} \quad g(x) = \frac{x - 1}{x^2 + 1}, \quad h(x) = 2x - 3$$

Differentiating w.r.t. "x", we get

$$f^{|}(x) = \frac{x}{\sqrt{x^2 + 1}}, \; g^{|}(x) = \frac{1 - 2x - x^2}{(x^2 + 1)^2}, \; h^{|}(x) = 2$$

$$f^{|}(h^{|}(g^{|}(x))) = \frac{2}{\sqrt{5}}$$

21. LetA = IR – {3} and B = IR – {1}.

Consider the function f :A $\rightarrow$ B defined by $f(x) = \left(\dfrac{x - 2}{x - 3}\right)$. Show that fis one-one and onto and hence find f^{-1}.

Ans. Let $x_1, x_2 \in$ A and $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\therefore \quad x_1 x_2 - 2x_2 - 3x_1 = x_1 x_2 - 2x_1 - 3x_2$$

$$\Rightarrow x_1 = x_2$$

Hence f is 1–1

Let y $\in$ B, $\therefore$ $y = f(x)$

$$\Rightarrow y = \frac{x - 2}{x - 3} \Rightarrow xy - 3y = x - 2$$

$$\text{or } x = \frac{3y - 2}{y - 1}$$

since $y \neq 1$ and $\frac{3y - 2}{y - 1} \neq 3$, $x \in A$

Hence f is ONTO

and $f^{-1}(y) = \frac{3y - 2}{y - 1}$

22. Show that $f : N \to N$, given by

$f(x)\begin{cases} x + 1, \text{ if } x \text{ is odd} \\ x - 1, \text{ if } x \text{ is even} \end{cases}$ is both one-one and onto.

Ans. Let x_1 be odd and x_2 be even and suppose $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 - 1 \Rightarrow x_2 - x_1 = 2$ which is not possible

simlarly, if x_2 is odd and x_1 is even, not possible to have $f(x_1) = f(x_2)$

Let x_2 and x_2 be both odd $\Rightarrow f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

simlarly, if x_1 and x_1 are both even, then also $x_1 = x_2$

$\therefore f$ is one – one

Also, any odd number 2r + 1 in co-domain N is the image of (2r + 2) in domain N and any even number 2r in the co-domain N is the image of (2r – 1) in domain N

$\Rightarrow f$ is on to

23. A binary operation $*$ on the set {0, 1, 2, 3, 4, 5} is defined as :

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is, invertible with 6 – a, being the inverse of 'a'.

Ans. $\left.\begin{array}{l} \text{since } a * 0 = a + 0 = a \\ \text{and } 0 * a = 0 + a = a \end{array}\right\} \forall\, a \in \{0, 1, 2, 3, 4, 5\}$

$\therefore$ 0 is the identity for $*$.

Also, $\forall\, a \in \{0, 1, 2, 3, 4, 5\}$, $a * (6 - a) = a + (6 - a) - 6 = 0$ (which is identity)

$\therefore$ Each element 'a' of the set is invertible with (6 – a), being the inverse of 'a'.

24. 26. Let A = R – {1}. If $f : A \to A$ is a mapping defined by $f(x) = \frac{x - 2}{x - 1}$, show that f is bijective, find f^{-1}.

Also find : (i) x if $f^{-1}(x) = \frac{5}{6}$ (ii) $f^{-1}(2)$

Ans. $f : A \to A$

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 1} = \frac{x_2 - 2}{x_2 - 1}$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one

Now $y = \frac{x - 2}{x - 1} \Rightarrow x - 2 = xy - y$

$\Rightarrow x(y - 1) = y - 2$

$\Rightarrow x = \frac{y - 2}{y - 1}$

For each $y \in A = R - \{1\}$, there exists $x \in A$

Thus f is onto. Hence f is bijective

and $f^{-1}(x) = \frac{x - 2}{x - 1}$

(i) $f^{-1}(x) = \frac{5}{6} \Rightarrow \frac{x - 2}{x - 1} = \frac{5}{6} \Rightarrow x = 7$

(ii) $f^{-1}(2) = 0$

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